

Electric Potential

Equipotential Surfaces; Potential due to a Point Charge and a Group of Point Charges; Potential due to an Electric Dipole; Potential due to a Charge Distribution; Relation between Electric Field and Electric Potential Energy

1. How to find the electric force on a particle 1 of charge q_1 when the particle is placed near a particle 2 of charge q_2 .
2. A nagging question remains: How does particle 1 “know” of the presence of particle 2?
3. That is, since the particles do not touch, how can particle 2 push on particle 1—how can there be such an *action at a distance*?
4. One purpose of physics is to record observations about our world, such as the magnitude and direction of the push on particle 1. Another purpose is to provide a deeper explanation of what is recorded.
5. One purpose of this chapter is to provide such a deeper explanation to our nagging questions about electric force at a distance. We can answer those questions by saying that particle 2 sets up an electric field in the space surrounding itself. If we place particle 1 at any given point in that space, the particle “knows” of the presence of particle 2 because it is affected by the electric field that particle 2 has already set up at that point. Thus, particle 2 pushes on particle 1 not by touching it but by means of the electric field produced by particle 2. One purpose of physics is to record observations about our world,

Equipotential Surfaces:

Potential energy:

When an electrostatic force acts between two or more charged particles within a system of particles, we can assign an electric potential energy U to the system.

If the system changes its configuration from an initial state \mathbf{i} to a different final state \mathbf{f} , the electrostatic force does work W on the particles. The change ΔU in the potential energy of the system is

$$\Delta U = U_f - U_i = -W \quad (1)$$

The work done by the electrostatic force is path independent. The work W done by the force on the particle is the same for all paths between points \mathbf{i} and \mathbf{f} .

Also, we usually set the corresponding reference potential energy to be zero. Suppose that several charged particles come together from initially infinite separations (state \mathbf{i}) to form a system of neighboring particles (state \mathbf{f}).

Let the initial potential energy U_i be zero, and let W_f represent the work done by the electrostatic forces between the particles during the move in from infinity. Then, the final potential energy U of the system is

$$U = -W_\infty$$

Electric Potential:

An electric potential (also called the *electric field potential*, potential drop or the **electrostatic potential**) is the amount of work needed to move a unit of positive charge from a reference point to a specific point inside the field without producing acceleration.

The **electric potential**, or **voltage**, is the difference in potential energy per unit charge between two locations in an electric field.

Electric potential, the amount of work needed to move a unit charge from a reference point to a specific point against an electric field. Typically, the reference point is Earth, although any point beyond the influence of the electric field charge can be used.

The potential energy of a charged particle in an electric field depends on the charge magnitude.

For an example, suppose we place a test particle of positive charge 1.60×10^{-19} C at a point in an electric field where the particle has an electric potential energy of 2.40×10^{-17} J. Then the potential energy per unit charge is

$$(2.40 \times 10^{-17} \text{ J}) / (1.60 \times 10^{-19} \text{ C}) = 150 \text{ J/C}$$

Next, suppose we replace that test particle with one having twice as much positive charge, 3.20×10^{-19} C. We would find that the second particle has an electric potential energy of 4.80×10^{-17} J, twice that of the first particle.

However, the potential energy per unit charge would be the same, still 150 J/C.

The potential energy per unit charge at a point in an electric field is called the electric potential V (or simply the potential) at that point. Thus,

$$V = U/q \quad (1)$$

Note that electric potential is a scalar, not a vector.

The electric potential difference ΔV between any two points i and f in an electric field is equal to the difference in potential energy per unit charge between the two points:

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} \quad (2)$$

Put $-W$ for ΔU in above equation

$$\Delta V = V_f - V_i = -\frac{W}{q} \quad (3)$$

The potential difference between two points is thus the negative of the work done by the electrostatic force to move a unit charge from one point to the other. A potential difference can be positive, negative, or zero, depending on the signs and magnitudes of q and W .

If we set $U_i = 0$ at infinity as our reference potential energy, then by Eq. 1, the electric potential V must also be zero there. Then from Eq. 3, we can define the electric potential at any point in an electric field to be

$$V = -\frac{W_{\infty}}{q} \quad (4)$$

Work Done by an Applied Force

Suppose we move a particle of charge q from point i to point f in an electric field by applying a force to it. During the move, our applied force does work W_{app} on the charge while the electric field does work W on it. By the work - kinetic energy theorem, the change ΔK in the kinetic energy of the particle is

$$\Delta K = K_f - K_i = W_{\text{app}} + W. \quad (1)$$

Now suppose the particle is stationary before and after the move. Then K_f and K_i are both zero, and Eq. 1 reduces to

$$W_{\text{app}} = -W. \quad (2)$$

The work W_{app} done by our applied force during the move is equal to the negative of the work W done by the electric field - provided there is no change in kinetic energy.

we can relate the work done by our applied force to the change in the potential energy of the particle during the move.

As we know that $\Delta U = U_f - U_i = -W$, so

$$\Delta U = U_f - U_i = W_{\text{app}} \quad (3)$$

We can relate our work W_{app} to the electric potential difference ΔV between the initial and final locations of the particle.

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

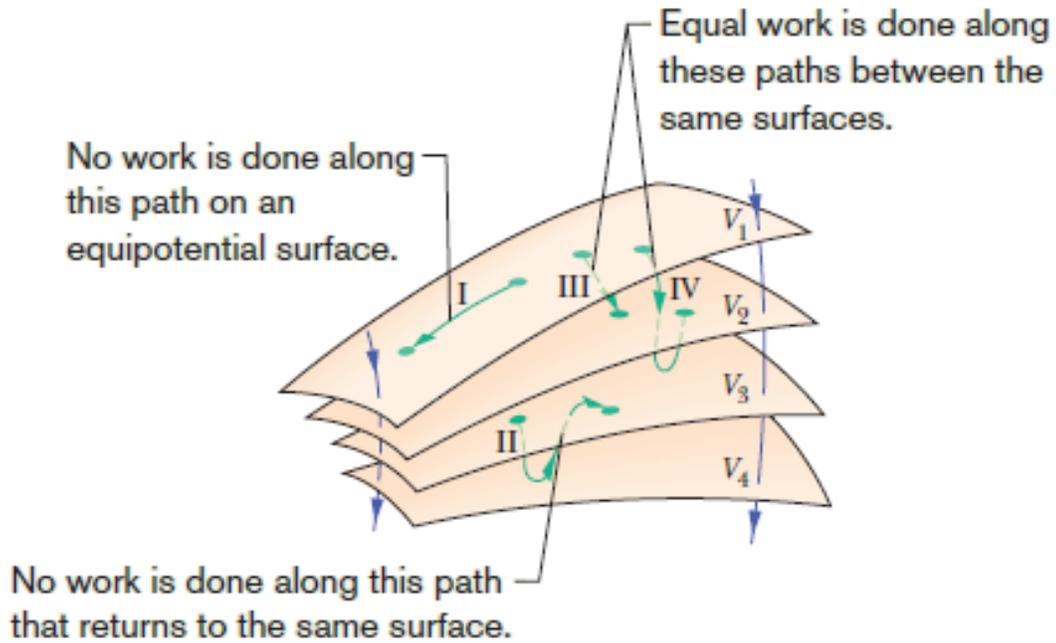
$$W_{\text{app}} = q \Delta V.$$

Equipotential Surfaces

Adjacent points that have the same electric potential form an **equipotential surface**, which can be either an imaginary surface or a real, physical surface. No net work W is done on a charged particle by an electric field when the particle moves between two points i and f on the same equipotential surface.

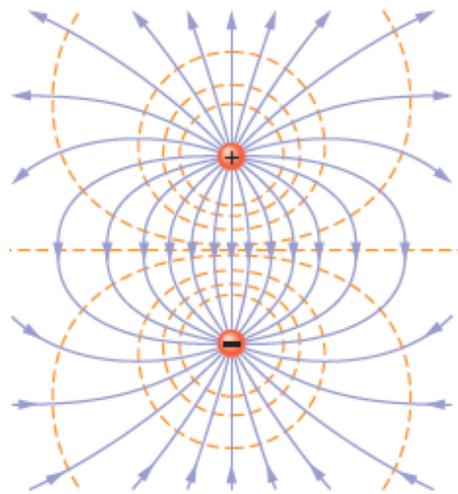
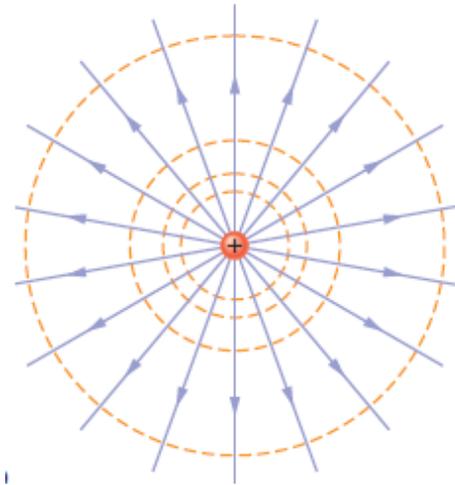
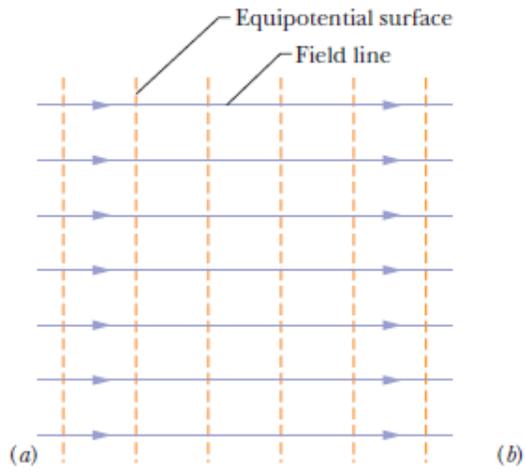
W must be zero if $V_f = V_i$.

Figure 1 below shows a family of equipotential surfaces associated with the electric field due to some distribution of charges. The work done by the electric field on a charged particle as the particle moves from one end to the other of paths **I** and **II** is zero because each of these paths begins and ends on the same equipotential surface and thus there is no net change in potential. The work done as the charged particle moves from one end to the other of paths **III** and **IV** is not zero but has the same value for both these paths because the initial and final potentials are identical for the two paths; that is, paths **III** and **IV** connect the same pair of equipotential surfaces.



From symmetry, the equipotential surfaces produced by a point charge or a spherically symmetrical charge distribution are a family of concentric spheres. For a uniform electric field, the surfaces are a family of planes perpendicular to the field lines. In fact, equipotential surfaces are always perpendicular to electric field lines and thus to \vec{E} , which is always tangent to these lines. If \vec{E} were not perpendicular to an equipotential surface, it would have a component lying along that surface. This component would then do work on a charged particle as it moved along the surface. However, by Eq. $\Delta V = V_f - V_i = -w/q$ work cannot be done if the surface is truly an equipotential surface; the only possible conclusion is that \vec{E} must be everywhere perpendicular to the surface.

Figure 2 shows electric field lines and cross sections of the equipotential surfaces for a uniform electric field and for the field associated with a point charge and with an electric dipole.

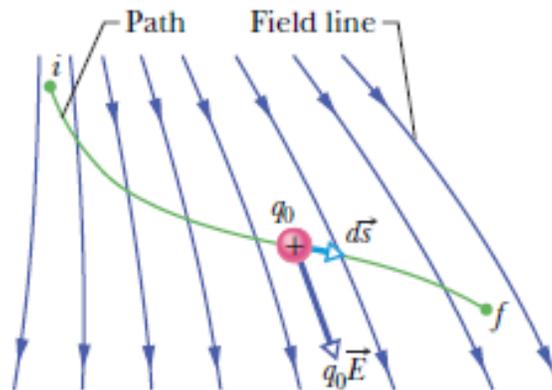


Electric field lines (purple) and cross sections of equipotential surfaces (gold) for (a) a uniform electric field, (b) the field due to a point charge, and (c) the field due to an electric dipole.

Calculating the Potential from the Field

We can calculate the potential difference between any two points i and f in an electric field if we know the electric field vector \mathbf{E} all along any path connecting those points.

To make the calculation, we find the work done on a positive test charge by the field as the charge moves from i to f .



Consider an arbitrary electric field, represented by the field lines in Fig. above, and a positive test charge q_0 that moves along the path shown from point i to point f .

At any point on the path, an electrostatic force $q_0\mathbf{E}$ acts on the charge as it moves through a differential displacement.

We know that the differential work dW done on a particle by a force \mathbf{F} during a displacement $d\mathbf{s}$ is given by the dot product of the force and the displacement:

$$dW = \vec{F} \cdot d\vec{s}.$$

$$dW = q_0\vec{E} \cdot d\vec{s}.$$

To find the total work W done on the particle by the field as the particle moves from point i to point f , the differential works done on the charge as it moves through all the displacements along the path:

$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}.$$

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

As we know that

So the above equation become

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

Above equation allows us to calculate the difference in potential between any two points in the field. If we set potential $V_i = 0$, then it becomes

$$V = -\int_i^f \vec{E} \cdot d\vec{s},$$

Potential Due to a Point Charge

To derive relation ΔV for the space around a charged particle, an expression for the electric potential V relative to the zero potential at infinity.

Consider a point P at distance R from a fixed particle of positive charge q (Fig. blow).

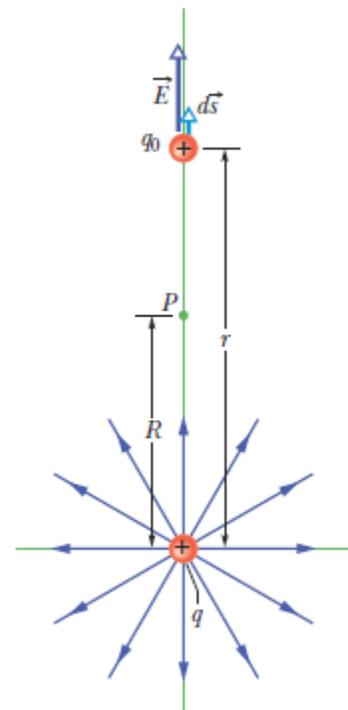
We move a positive test charge q_0 from point P to infinity.

Because the path we take does not matter, let us choose the simplest one -- a line that extends radially from the fixed particle through P to infinity.

Evaluate the dot product

$$\vec{E} \cdot d\vec{s} = E \cos \theta ds.$$

The electric field in Fig. is directed radially outward from the fixed particle. Thus, the differential displacement $d\vec{s}$ of the test particle along its path has the same direction as \vec{E} . That means that in Eq. above, angle $\theta = 0$ and $\cos \theta = 1$. Because the path is radial, let us write $d\vec{s}$ as $d\vec{r}$. Then, substituting the limits R and ∞ , we can write as



$$V_f - V_i = -\int_R^\infty E dr.$$

Next, we set $V_f = 0$ (at ∞) and $V_i = V$ (at R). Then, for the magnitude of the electric field at the site of the test charge, we substitute from Eq.

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}.$$

The above equation becomes

$$\begin{aligned} 0 - V &= -\frac{q}{4\pi\epsilon_0} \int_R^\infty \frac{1}{r^2} dr = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} \right]_R^\infty \\ &= -\frac{1}{4\pi\epsilon_0} \frac{q}{R} \end{aligned}$$

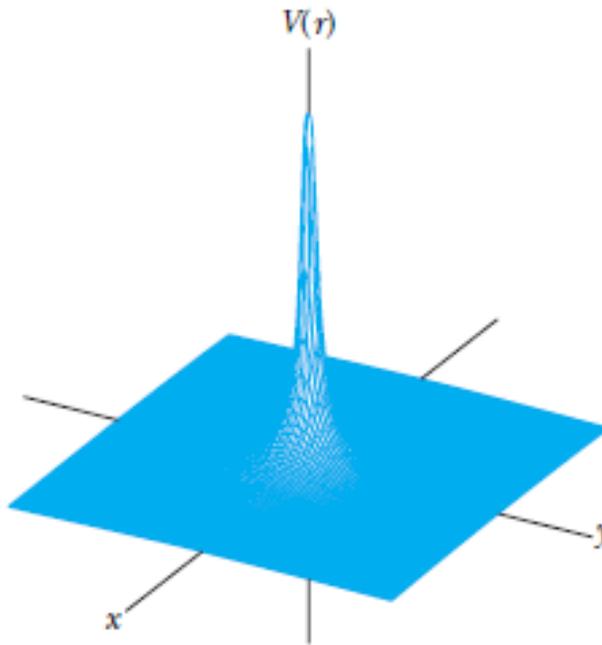
Solving for V and switching R to r , we then have

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

as the electric potential V due to a particle of charge q at any radial distance r from the particle.

A computer-generated plot of the electric potential $V(r)$ due to a positive point charge located at the origin of a xy plane.

The potentials at points in the xy plane are plotted vertically. (Curved lines have been added to help you visualize the plot.) The infinite value of V predicted by above eq. for $r = 0$ is not plotted.



Potential Due to a Group of Point Charges

We can find the net potential at a point due to a group of point charges with the help of the superposition principle.

Using Eq. $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ with the sign of the charge included, we calculate separately the potential resulting from each charge at the given point. Then we sum the potentials. For n charges, the net potential is

$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i} \quad (n \text{ point charges}).$$

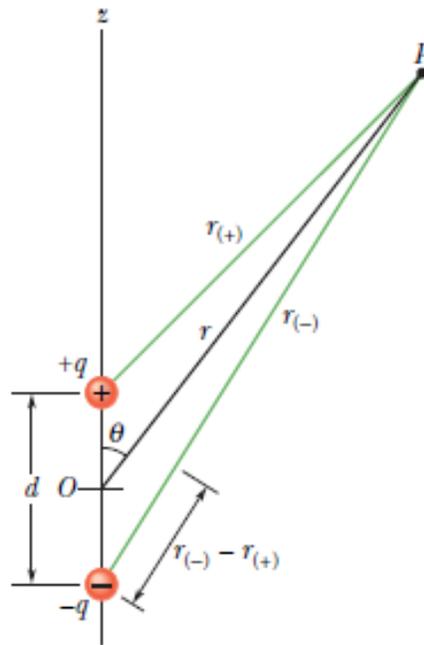
Here q_i is the value of the i^{th} charge and r_i is the radial distance of the given point from the i^{th} charge.

The sum in above Eq. is an algebraic sum, not a vector sum like the sum that would be used to calculate the electric field resulting from a group of point charges.

Here in lies an important computational advantage of potential over electric field: It is a lot easier to sum several scalar quantities than to sum several vector quantities whose directions and components must be considered.

Potential Due to an Electric Dipole

To find the potential at an arbitrary point P in Fig of to an electric dipole.



At P , the positive point charge (at distance $r_{(+)}$) sets up potential $V_{(+)}$ and the negative point charge (at distance $r_{(-)}$) sets up potential $V_{(-)}$. Then the net potential at P is given by

$$V = \sum_{i=1}^2 V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

Naturally occurring dipoles - such as those possessed by many molecules - are quite small; so we are usually interested only in points that are relatively far from the dipole, such that $r \gg d$ where

d is the distance between the charges. Under those conditions, the approximations that follow from Fig.

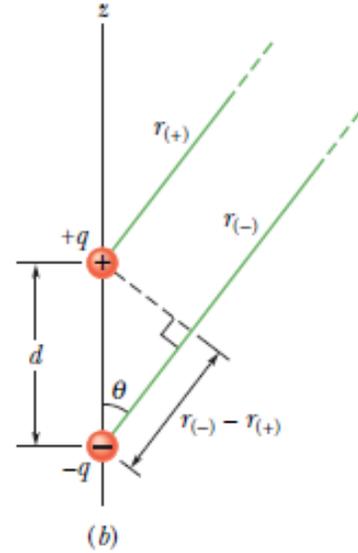
$$r_{(-)} - r_{(+)} \approx d \cos \theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

Put these value in above equation, we get

$$V = \frac{q}{4\pi\epsilon_0} \frac{d \cos \theta}{r^2},$$

where θ is measured from the dipole axis as shown in Fig. a. We can now write V as

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2} \quad (\text{electric dipole}),$$



in which $p (= qd)$ is the magnitude of the electric dipole moment \mathbf{P} . The vector \mathbf{P} is directed along the dipole axis, from the negative to the positive charge. (Thus, θ is measured from the direction of \mathbf{P} .) We use this vector to report the orientation of an electric dipole.

Potential Due to a Continuous Charge Distribution

When a charge distribution q is continuous (as on a uniformly charged thin rod or disk), we cannot use the summation to find the potential V at a point P .

Instead, we must choose a differential element of charge dq , determine the potential dV at P due to dq , and then integrate over the entire charge distribution.

Let us again take the zero of potential to be at infinity. If we treat the element of charge dq as a point charge, then we can express the potential dV at point P due to dq :

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

Here r is the distance between P and dq . To find the total potential V at P , we integrate to sum the potentials due to all the charge elements:

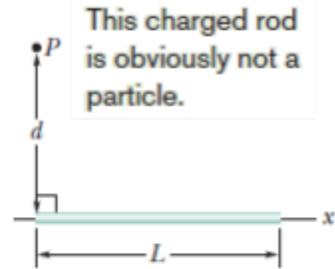
$$V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

The integral must be taken over the entire charge distribution. Note that because the electric potential is a scalar, there are no vector components to consider.

We now examine two continuous charge distributions, a line and a disk.

Line of Charge

In Fig.(a), a thin nonconducting rod of length L has a positive charge of uniform linear density λ . Let us determine the electric potential V due to the rod at point P , a perpendicular distance d from the left end of the rod.

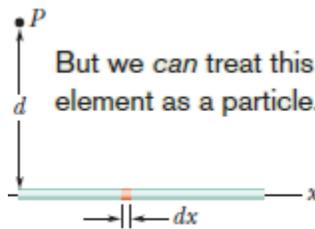


(a)

We consider a differential element dx of the rod. This element of the rod has a differential charge of

$$dq = \lambda dx.$$

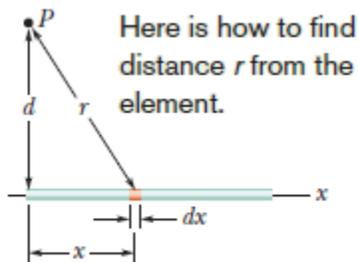
A thin, uniformly charged rod produces an electric potential V at point P



(b)

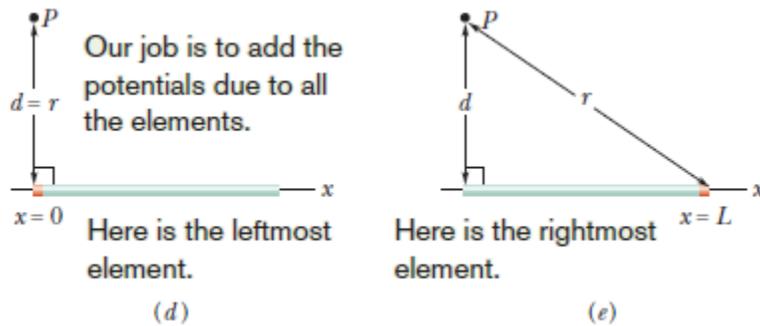
An element can be treated as a particle.

This element produces an electric potential dV at point P , which is a distance $r = (x^2 + d^2)^{1/2}$ from the element (Fig. c).



(c)

The potential at P due to the element depends on the distance r



We need to sum the potentials due to all the elements, from the left side (d) to the right side (e).

Treating the element as a point charge, we can write the potential dV as charge, we can write the potential dV as

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(x^2 + d^2)^{1/2}}$$

Since the charge on the rod is positive and we have taken $V = 0$ at infinity

We now find the total potential V produced by the rod at point P by integrating above Eq. along the length of the rod, from $x = 0$ to $x = L$ (Figs. d and e), using integral 17 in Appendix E. We find

$$\begin{aligned}
V &= \int dV = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda}{(x^2 + d^2)^{1/2}} dx \\
&= \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}} \\
&= \frac{\lambda}{4\pi\epsilon_0} \left[\ln\left(x + (x^2 + d^2)^{1/2}\right) \right]_0^L \\
&= \frac{\lambda}{4\pi\epsilon_0} \left[\ln\left(L + (L^2 + d^2)^{1/2}\right) - \ln d \right].
\end{aligned}$$

We can simplify this result by using the general relation $\ln A - \ln B = \ln(A/B)$.

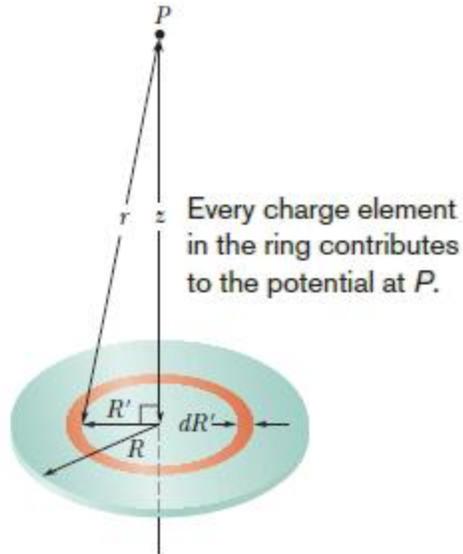
We then find

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{L + (L^2 + d^2)^{1/2}}{d} \right]$$

Because V is the sum of positive values of dV , it too is positive, consistent with the logarithm being positive for an argument greater than 1.

Charged Disk

We calculated the magnitude of the electric field at points on the central axis of a plastic disk of radius R that has a uniform charge density σ on one surface. Here we derive an expression for $V(z)$, the electric potential at any point on the central axis.



In Fig., consider a differential element consisting of a flat ring of radius R' and radial width dR' . Its charge has magnitude

$$dq = \sigma(2\pi R')(dR')$$

in which $(2\pi R')(dR')$ is the upper surface area of the ring. All parts of this charged element are the same distance r from point P on the disk's axis. With the aid of Fig., we can write the contribution of this ring to the electric potential at P as

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}$$

We find the net potential at P by adding (via integration) the contributions of all the rings from $R' = 0$ to $R' = R$:

$$V = \int dV = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{R' dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z)$$

Note that the variable in the second integral of Eq. is R' and not z , which remains constant while the integration over the surface of the disk is carried out. (Note also that, in evaluating the integral, we have assumed that $z \geq 0$.)